Information Entropy Theory and Asset Valuation: A Literature Survey

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Abstract

The purpose of this study is to review the empirical work applied to market efficiency, portfolio selection and asset valuation, focusing on the presentation of the comprehensive theoretical framework of Information Entropy Theory (IET). In addition, we examine how entropy addresses the shortcomings of traditional models for valuing financial assets, including the market efficiency hypothesis, the capital asset pricing model (CAPM), and the Black and Scholes option pricing model. We thoroughly reviewed the literature from 1948 to 2022 to achieve our objectives, including well-known asset pricing models and prominent research on information entropy theory. Our results show that portfolio managers are particularly attracted to valuations and strive to achieve maximum returns with minimal risk. The entropy-based portfolio selection model outperforms the standard model when return distributions are non-Gaussian, providing more comprehensive information about asset and distribution probabilities while emphasising the diversification principle. This distribution is then linked to the entropic interpretation of the no-arbitrage principle, especially when extreme fluctuations are considered, making it preferable to the Gaussian distribution for asset valuation. This study draws important conclusions from its extensive analysis. First, entropy better captures diversification effects than variance, as entropy measures diversification effects more generically than variance. Second, mutual information and conditional entropy provide reasonable estimates of systematic and specific risk in the linear equilibrium model. Third, entropy can be used to model non-linear dependencies in stock return time series, outperforming beta in predictability. Finally, information entropy theory is strengthened by empirical validation and alignment with financial views. Our findings enhance the understanding of market efficiency, portfolio selection and asset pricing for investors and decision-makers. Using Information Entropy Theory as a theoretical framework, this study sheds new light on its effectiveness in resolving some of the limitations in traditional asset valuation models, generating valuable insights into the theoretical framework of the theory.

Keywords: *Information entropy theory; asset valuation; Capital Asset Pricing Model(CAPM); Diversification; Gaussian Distribution.*

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1. Introduction

Investments in the financial market are guided by optimisation programs seeking to maximise expected returns while minimising risk, popularised by Markowitz's mean-variance model (1952). This risk management approach's underlying assumption is that financial agents make rational decisions in a risky universe. In addition, markets are efficient, as Fama (1970) argued, implying that a security's fundamental value equals its market value, irrespective of future returns.

Entropy has been utilised in portfolio selection theory to measure diversity, with several studies examining market efficiency, such as George et al. (2021), Gong et al. (2022), Gu et al. (2021), Horta et al. (2014), MacLean et al. (2022), Mahmoud and Naoui (2017), Sukpitak and Hengpunya (2016), Zhao et al. (2020), and Zhou et al. (2013).

To solve the asset selection problem, Markowitz (1952) developed the mean-variance method. Sharpe's (1964) CAPM model became the standard for valuing financial assets. According to Fama (1970), market efficiency is essential to classical finance models. Prices instantly absorb all relevant fundamental information in an efficient market of rational investors. As a result, investors seek to maximise their profits without beating the market (Fama, 1970). Black and Scholes (1973) contributed significantly to financial theory, developing a model for option pricing.

Options are valued chiefly by the underlying asset's value, unlike stocks, bonds, or other financial assets. Option valuation has been based on this model for many years. According to traditional asset pricing models (Carhart, 1997; Fama & French, 1993, 2015; Ross, 1976; Sharpe, 1964), observed asset returns reflect observed risks.

A crucial point to note here is that the classical valuation of assets (Fama, 1970; Markowitz, 1952; Sharpe, 1964) derives from Black and Scholes (1973), who measured stock likelihood through the variance and mean of Gaussian probability distributions.

Throughout the 1960s, work in market finance contributed to developing models for valuing economic assets. The rationality hypothesis underlies these models, which advocate efficient financial markets and provide portfolio managers with tools to manage their portfolios. This vision focuses on averages, stability, and the return to equilibrium following a shock. These models revolve around a hypothesis narrow hypothesis: all markets in a Gaussian universe are efficient (Markowitz, 1952; Fama, 1970; Sharpe, 1964).

Information Entropy Theory (IET), based on (Backus et al.'s (2014) theory, mathematically and physically reduces entropy. This theory provides an innovative and promising approach to market efficiency, portfolio selection and financial asset valuation. IET can be a powerful tool for portfolio selection and asset pricing, with applications to understanding financial market dynamics and functioning. It has been used in various studies, including those by Li and Liu (2008) and Xu et al. (2011).

As defined by Shannon (1948), the information entropy theory measures portfolio returns. Various methods have been developed to measure portfolio risk (Xu et al., 2011) and evaluate option prices (Gulko, 1999). Risk and return are intrinsically interconnected, necessitating a trade-off between entropy and variance.

In addition, entropy theory can provide novel insights into market behaviour. In Information Entropy Theory (IET), behavioural and informational investment theories are integrated within a common framework, contributing to a better understanding investors' decision-making processes.

In their study, Backus et al. observed that. It has been demonstrated that the information entropy theory can be applied to asset pricing models. It has also been suggested that Breuer and Csiszár (2013), as well as Piquet et al. (2021) and Ormos and Zibriczky (2014) should be used to assess the uncertainty surrounding portfolio selection in terms of performance, Brissaud (2005) to quantify accounting information loss, and Breuer and Csiszár (2013). Further, Hancock et al. (2011) have utilised entropy theory to measure how human decision-making in financial matters is influenced by entropy.

Further, Piquet et al. (2021) introduced the concept of a fuzzy LR power number to describe risk asset return rates to accommodate different investor attitudes about risk assets. To diversify portfolios without incurring additional risks, their model incorporates a risk-free constraint which allocates downside risk based on the individual's risk attitude when allocating downside risk (Zhao et al., 2020). MPSOs are superior to traditional PSOs in a real-world setting in optimising multi-particle problems using swarm optimisation methods. These have proven advantageous for investors with different risk attitudes based on developing swarm optimisation methods for multi-particle problems. As a point of caution, it is worth noting that the model does not consider short sales, which may be an exciting consideration for future research.

This study reviews the empirical work applied to market efficiency, portfolio selection and asset valuation to expose the comprehensive theoretical framework of Information Entropy Theory (IET). Further, we investigate how entropy treats traditional models' shortcomings for valuing financial assets (market efficiency hypothesis, CAPM, Black and Sholes option pricing model).

This study aims to contribute to the IET literature in several ways. In addition to justifying the efficient market hypothesis using the entropy hypothesis, entropy-based portfolio selection models emphasise diversification, correct Black-Scholes' equation by adding adjusted volatility, and introduce redundancy to solve theoretical problems.

We conclude that (a) Entropy is a more accurate measure of risk than diversification variance. In contrast to variance, entropy applies to diversification's effects more broadly. (b) With mutual information and conditional entropy, the linear equilibrium model estimates systematic and specific risk well. (c) Entropy can be used to model non-linear dependencies regarding predictability. (d) The asset pricing model's entropy is more potent than beta. Empirical validations support this information entropy theory and correspond to financial perspectives. Furthermore, our findings enhance investor and decision-maker understanding of most major empirical finance studies.

This paper is divided into sections. Section 2 reviews traditional asset valuation methods and discusses their shortcomings. The effectiveness of the IET's risk measurement system is assessed. Our findings are presented and analysed in section 3. Section 4 of the study highlights information entropy.

2. Methodology

We completed a comprehensive literature review covering 1948 to 2022 to conduct this study. This study aims to collect empirical evidence and academic papers on market efficiency, portfolio selection, asset valuation and risk measurement. Academic databases, journals and relevant publications will be searched. Using existing research as a basis for our analysis, we can draw valuable insights and implications for future applications of IET in the financial sector. This is based on our analysis.

To ensure the accuracy and reliability of our review, we will use a systematic and rigorous approach to selecting relevant literature from a wide range of sources. We will critically evaluate each study's methodology and conclusions as part of our analysis. We will also compare and contrast views and arguments about IET's effectiveness in addressing traditional models' limitations. This project will also improve asset valuation and risk measurement practices.

In summary, the methodology section of this paper outlines our approach to examining the shortcomings of traditional asset valuation models and making suggestions for improvement. A basic understanding of Information Entropy Theory's theoretical framework is explored. In addition, the theory applies to asset valuation and risk assessment. As a result of our study, we believe that IET can improve financial decision-making and contribute to a better understanding of market behaviour and uncertainty in asset pricing.

2.1 Shortcomings of Traditional Asset Valuation Models

According to efficiency theory, assets trade at a price that reflects all information instantly (Fama, 1965). Unless additional information is provided, the price will stay the same. Therefore, each price in the future is independent of the previous one. Additionally, we exist in a situation where information influences prices in a weakly interdependent manner. Because of this, we live in a world of chance. According to Sharpe (1963), investment returns are exclusively related because they are market-based.

Portfolio theory can benefit from the utility of risk, risk aversion, risk measurement, and expectation-variance approaches (Sharpe, 1963). Portfolio theory solves economic decision problems under threat as investors choose financial investments with random returns according to this theory (Markovitz, 1959). Securities are not included in this study's portfolio theory of financial assets, and various guarantees are available on the market.

Numerous articles have reviewed the literature on capital asset pricing models (CAPMs). Despite Fama and French's (2015) findings, CAPM does not appear responsible for anomalous models. A wide range of works on stock market anomalies has been cited by Fama and French (1993). The authors explain stock returns by book-to-market equity and size. By removing the most extreme 1% of observations per month, Knez and Ready (1997) deny that size and book-to-market equity are combined to explain the size risk premium.

In response to criticism of the CAPM, Ross (1976) lodged Arbitrage Pricing Theory (APT). APT emphasises that expected return is determined by several influencing variables instead of a single risk factor. Unlike the CAPM, the APT is not affected by the problems of measuring market exposure. It is commonly assumed that market prices reflect all relevant information instantly, which is one of the main assumptions of asset price models (Ross, 1976). The validity of this hypothesis has been the main subject of debate, and many studies have been conducted to test it against actual market data. Although the efficient market hypothesis is not universally accepted, its simplicity makes it essential. It allows for the handling of increasingly complex models of asset pricing theory. Since then, financial theory has struggled to identify the factors that explain investment performance (Hübner et al., 2015).

Different perspectives on asset pricing have been explored in the literature, including the CAPM. Assuming market efficiency, the CAPM determines the risk price. Risk and return are therefore traded off in it. Based on the CAPM, equity returns are determined by market returns and the covariance of interest rates. The beta coefficient measures the return on stock investment as a function of its regression slope. Market betas are, therefore, a positive linear function of expected stock returns. Cross-sectional stock returns, however, can be explained by market betas.

It is impossible to create a perfect market, and investment would be more attractive if prices accurately reflect all information (Grossman & Stiglitz, 1980). Therefore, Market efficiency is measured over time to determine how it has changed. To explain variations in market efficiency over time, An adaptive market hypothesis (AMH) has been proposed by Lo (2004). Market efficiency no longer varies across time or space due to the revised AMH hypothesis.

Ex-ante risk premia and betas are incorporated into the CAPM. Time series data are used to calculate excess return rates and betas. However, betas and risk awards for individual assets change. t is implicitly assumed that betas and average asset returns are consistent in time series analysis because of the two-period structure of the CAPM. Thirdly, many assets are non-tradable, and the CAPM tests are invariably based on market portfolio proxies that exclude significant asset classes such as human capital. Literature has utilised different approaches to address these issues.

A study by Fama and French (1993) found that stock returns are correlated with firm characteristics, such as the ratio of cash flow to price, the earnings ratio to cost, and price-to-book values. Anomalies in CAPM prevent CAPM from explaining these elements. As a result, Fama and French (1996) developed the three-factor model to describe them. Rosenberg et al. (1985) constitute inefficiency indicators which raises questions about the joint hypothesis of informational efficiency as the risk is a linear combination of several factors (Fama & French, 2015). Therefore, it is not anomalies that the ex-post CAPM does not capture but premiums that compensate for the asset's risk. According to Fama and French (1996), the solution aims to introduce a novel dimension to the asset risk equation. According to Campbell et al. (1997), total efficiency is impossible.

The CAPM fails empirical tests, implying that most applications are invalid (Fama & French, 2015). Risk can be measured using returns variance in models. It is hypothesised that returns should have a normal distribution or be distributed according to any two-parameter distribution of returns. It is more appropriate to use a measure of risk that is consistent with the preferences of active and potential shareholders (such as consistent risk measures). However, this re about the probability of losing: it is asymmetrical (K. D. Daniel et al., 2001). Fama and French's (1993) model extends Carhart's (1997) model. Sharpe (1964), Mossin (1966), Fama and French (1993), and Daniel and Titman (1999) added the Momentum factor in 1993 as described by Sharpe (1964), alongside the CAPM used by Carhart (1997). Investing in momentum means choosing stocks that have performed well and are likely to do so in the future.

While Kothari and Warner (2001) found that the Fama-French and Carhart models provided better results, the classical CAPM has disadvantages. The paper to detect the abnormal significance of the result does not exist. Other criticisms are still valid. Three major conceptual problems need to be addressed to test the CAPM. Developing returns under the CAPM imposes non-linear difficulties on the economic model. When stretching whether a market proxy portfolio lies on the portfolio frontier, betas, variances, and covariances must be estimated with this constraint. However, this depends on the parameters that are to be evaluated. Rather than measuring systematic risk, Sharpe's (1966) index measures the portfolio's performance, not individual assets. Jensen (1978) and Treynor (1965) analyse portfolios and individual securities.

Several criticisms of Treynor's (1965) model are as follows. First, Treynor considers only systematic risk. Second, the index measures the ability of managers to select individual stocks but not their ability to predict market movements. Third, the index uses a risk-free investment rate that may vary from investment to investment. Fourth, the Sharpe index (1966) measures the variability of returns. Diversifying, but not separating, upside from downside risks is the aim of this measure. Risk-free return rates are equal to lending and borrowing rates. When is a risk level exceeded, which is invalid? This criticism applies to both Treynor and Sharpe's measures. Fifth, Treynor (1965) consider portfolios favourable only if they are above the securities market line.

At any given time, the price of an underlying asset can move up or down equally, according to Black and Sholes (1973). However, this is generally not true because many economic factors determine stock prices in the market. When they are wet, we cannot assign the same probability to their effect on asset price movements. According to Black and Sholes (1973), stock prices should be generally distributed. However, this hypothesis has been widely disputed and even rejected. There is a marked difference between empirical and Gaussian distributions regarding thick tails. As per the standard distribution definition, kurtosis coefficients are more significant than 3 when using the defined distribution.

A couple of exciting phenomena were observed by Rubinstein (1994). Until the October 1987 market crash, Black-Scholes appeared to be a good model for valuing S&P 500 options. He also said the Black-Scholes model has consistently undervalued ace-in-the-hole options since October 1987. In practitioners' terms, volatility smiles refer to patterns of mispricing. As strike prices increased, the implied volatility of the index for option money decreased. Even when volatility appears stable for short periods, it is never constant. Economic and political factors contribute to volatility, so it varies over time and is often accompanied by non-stationarity. The model assumes stable interest rates, and in reality, this hypothesis is untrue. In this model, the risk-free rate represents this constant and known rate.

2.2 IET and asset valuation

According to Shannon (1948), the essential objective of information theory is to measure information by defining a quantity of information. A source of entropy will be used.

Information is permanently attached to an event; the more unpredictable it is, the more data it contains**.** As an inference method rather than a physical theory, statistical mechanics can be seen as a form of statistical inference (Jaynes, 1957). To determine the partition function, we must follow the usual computational rules established by this maximum entropy principle, starting with choosing the partition function. Generally, time series with higher entropy values are less predictable and have better information transmission (Gulko, 1999).

A novel formula for pricing European options was introduced by Gulko (2002) based on the IET. He developed a beta model. , he discussed option replication and beta model properties. In many ways, the beta model is easier to use than the Black-Scholes model. Alternative option valuation models demonstrate fewer restrictions and greater accuracy than the beta model.

Different types of entropy are included in various concepts of entropy:

Shannon's Entropy (1948) expresses the amount of information available. As a measure of the uncertainty of a random event, it can be viewed as a measure of randomness. Specifically, the uncertainty function of an upcoming event is a function of its distribution. This is the information provided by that recent event, i.e., its uncertainty function. Shannon entropy offers unprecedented insight into the relationship between knowledge and thermodynamics. It could also be applied in any context where probabilities can be defined; it was not limited to thermodynamics. It is possible to consider thermodynamic entropy as a particular case of Shannon entropy since it is used to measure the chances of each state in an entire state space.

To develop an informal entropy theory, Shannon conducted statistical analyses based on Hartley's formula (1928) and developed his entropy measures. There has been much debate over the choice of various information measures, as Campbell and Thompson (2008) have investigated. The following steps were outlined during his presentation: characterisation of Shannon's entropy measure, characterisation of Shannon's estimation of Shannon's entropy of generalised probability distributions, description of the amount of information I(QI(P) and proving Markov chains' limit theorem based on information theory.

Claude Elwood Shannon, an engineer working at Bell Laboratories at the time and author of the book A Mathematical Theory of Communication published in 1948, published A Mathematical Theory of Communication. The following year, a re-publication of the work titled "The Mathematical Theory of Communication" was published under "The Mathematical Theory of Communication" to reach a wider audience, including an introduction by Warren Weaver. Shannon's theory of information is, therefore, also a theory of details by compression, which instead of considering any sequences, assumes that the lines transmitted verify specific statistical properties. Finally, Shannon's theory is about information content relative to a compression goal and to a particular statistical distribution of sequences. It is, therefore, not a limited theory of information because it only deals with the transmission; it is a theory of probabilistic information compatible with the algorithmic theory of knowledge and limited simply because it is relative to particular probabilistic distributions.

The term "entropy" was first used by Clausius (1854) to describe a quantity whose quantity increases with increasing heat levels. There is a reason why at that time (as it is still today), thermodynamic entropy is also called Clausius entropy. It is intended to pay homage to the one who invented the German term "entropy" by analogy to the word "energy" from the radical Greek meaning "action of transformation", as well as the person who also invented the English term "energy". There were no atomic structures of matter when the concept of thermodynamics was developed, and it only dealt with macroscopic quantities like temperature, pressure, and volume at the time.

Boltzmann (1872) associates the value and direction of a particle's velocity with coordinates corresponding to each speed at a point in mathematical space. The probability of a distribution is thus directly proportional to the number of microstates that realise it. Some distributions are more probable than others. The distributions are a set of system states, and the system spends more time in the form most likely to occur.

The Gibbs entropy of a classical macroscopic system is a function of the probability distribution in phase space. It applies to a set of techniques occupying the phase area’s whole or part. The question arises according to the nature of the probabilities taken into account in its calculation: if they are exclusively objective probabilities, there are no problem objective probabilities, there is no problem; if, on the other hand, we include in the formula with degrees of belief or with chances that have, for example, ignorance, and ignorance of the precise characteristics of the system and of the fluctuations (some researchers defend the possibility of mixing the two because it does not change anything once it is a question of measurement and expected value), so since the data are at least partially subjective, the at least somewhat personal, entropy must, if it still means anything, be itself emotional. Be true to yourself against the idea that entropy calculated with the Gibbs formula can carry a particular meaning of significance (Goldstein et al., 2020).

Approximate Entropy (APEN) is an approximate entropy of time series that Pincus (1991) proposed to quantify the randomness of time series. The "ApEn" value will increase when a time series data set has a high level of randomness. The approximate entropy of observations, called ApEn, can be described as a measure of statistical regularity to estimate the probability that a pattern of similar words will not be followed (S. Pincus & Singer, 1996).

Oh et al. (2007) used an integration dimension of m = two and a standard deviation of 20% for time series returns as an example of how this measure is applied to financial markets. A simple approximation to entropy is a calculation based on the probability that similar time series patterns will remain the same for subsequent comparisons. This is a simple calculation. Tests based on this method were designed to measure irregularities in a complex non-linear system. Still, Pincus and Kalman (2004) introduced them as a measure of market efficiency for stocks and foreign exchange.

Tsallis' entropy (1988) provides the same character as Shannon's entropy. The only difference is that for Tsallis entropy, the degree of homogeneity in a convex linearity condition is α instead of 1. The same formula was introduced by Havrda and Chárvat (1967) and Patil and Taillie (1982) in information theory to measure the biological diversity of organisms.

One of the easiest ways to calculate the distance between two points is to calculate the cross-entropy, as proposed by Kullback and Leibler (1951)**.** This measure measures how many bits are required to identify an event from a set of circumstances, calculated by looking at the cross-entropy between the two probability laws. Generally speaking, the term tribe is used in mathematics to describe the distribution of events based on a probability distribution q. This is relative to a reference distribution p on which the event distribution is found. There are differences between the Kullback cross-entropies and Tsallis relative entropies, and the difference between them is known as the Tsallis relative entropy.

Fuzzy entropy is an expression of fuzzy set theory, an origin of fuzzy entropy. The first definition of non-probabilistic entropy was given by De Luca and Termini (1972), who were influential in this development, as well as others such as Bhandari and Pal (1993), Kosko (1986), and Yager (2000). Li and Liu (2008) proposed a new definition of entropy based on it being characterised as uncertainty resulting from a lack of information caused by failure to predict precise values based on available data.

Proposed by Alfréd Rényi, Renyi entropy is a function that corresponds to the amount of information contained in the probability of the occurrence of a crisis (Jizba & Arimitsu, 2004). This is a random variable. Rényi entropy is used for communication and coding, data mining, detection, segmentation, classification, hypothesis testing, image alignment, etc.

The mean-entropy approach was compared to traditional methods: Construct all possible efficient portfolios from a randomly selected sample of monthly closing prices for 50 securities over 14 years. The mean-entropy portfolios were consistent with the Markowitz full-covariance and Sharpe single-index models.

Tabakis (2000) says two principles exist for choosing a risk-neutral measure among all entropy measures, which minimises the information each measure can obtain. These principles can also be called the maximal entropy principle of Gulko (1999). They used finite trading times and independent log returns to test this principle. As expected, this resulted in a distribution with exponential tails instead of Gaussian seats. In the presence of exponential tails, they implied volatility smiles. As for the second principle, Fisher's information minimises using a fixed measure, such as the Black-Scholes distribution, to compute probability distributions. This type of problem has been studied extensively using robust statistics, and the results are readily available. In the case of Gaussian central models, the Huber distribution, which has exponential tails, is the resulting distribution when Gaussian major models are employed.

The maximum entropy distribution of several samples was studied by Neri and Schneider (2012), who developed a simple and robust test for the maximum entropy distribution. The researchers then compare the results of their study with those of Buchen and Kelly. It is estimated that the full entropy distribution can be obtained by opting for the index option. According to their findings, Buchen and Kelly (1996) are on the same page. According to a growing number of research papers, the theory of portfolio selection is supported by entropy, and these studies include Smimou et al. (2007), Usta and Kantar (2011) and Xu et al. (2011). They propose several forms of generalised entropy. According to entropy theory, an optimal portfolio can be suggested according to a given probability of a return.

Using the entropic method, Brody et al. (2005) obtained the spot price dependence of options and the relevant Greeks within a time-reversed economy by applying the technique to the time-reversed economy. It is well known that entropic calibration has several advantages. To begin with, it can be ensured that all constraints are met precisely. There are arbitrage opportunities when the Lagrange multiplier root search does not converge, indicating mispricing and opportunities for arbitrage. Thirdly, there is the advantage of having a stable and fast algorithm to implement the result, which is the third benefit of the algorithm.

Based on Shannon's entropy, the author Usta and Kantar (2011) calculated portfolio diversity based on Shannon's entropy. Using probability as an objective function makes it possible to determine portfolio weights based on the probability function. Conservative investors preferred a portfolio that lacked short selling for theoretical and practical reasons. There has been considerable research on the theory of "mean-variance skewness entropy" to select portfolios. Using a novel denoised frequency domain entropy framework, Owusu Junior et al. (2021) then applied this framework to analyse global equity markets in the aftermath of the COVID-19 pandemic. Accordingly, they have argued the opposite theory to shock transmission: diversification benefits are derived from information flow.

Xu et al. (2011) use mixed entropy to estimate the risk associated with a random and fuzzy process based on an arbitrary and unclear approach. According to Usta and Kantar (2011), using a "mean-variance - skewness - entropy" model to test portfolio selection is more suitable than using traditional portfolio selection models in terms of its relevance. Using the objective entropy function to generate a diversified portfolio with optimal asset allocation, Borup et al. (2023) developed a new consumption-based model called the Revised CCAPM based on the objective entropy function.

By utilising Approximate Entropy, Bhaduri (2014) attempts to explain the stock market crash as it happened in three countries: the US, Japan, and India. Furthermore, the research team investigated the 1997 Asian crisis using weekly data from seven of the most critical Asian indices. As a result, Hong Kong, Malaysia, Singapore, Korea, Taiwan, Indonesia, and Japan are among the most influential. This study's vital signs point to a significantly reduced Apen level during these crashes, consistent with the critical symptoms.

Several conditions must be met to minimise the entropy martingale approach discussed and derived by Hunt and Devolder (2011). For such a measure to be effective, it should be arbitrage-free and attached to the martingale measure. It was found that Oh et al. (2015) used the return time series of several financial markets, such as the S&P500, KOSPI, and DAX, to examine the entropy density function and its variability over time. In the years since the subprime crisis, the S&P500 index has seen its entropy decrease significantly, while in the DAX and KOSPI markets, risk has not decreased considerably.

Sukpitak and Hengpunya (2016) examined the evolution over time of the Hurst exponent of the SET index based on the DFA method. There is also evidence that during the study period, the Hurst exponent tends to decrease to an ideal value of 0.5, which indicates an improvement in market efficiency, which suggests an increase in sales. Several factors influence the efficiency of the market, including market capitalisation. The mean-variance-skewness-kurtosis-entropy model was tested using various portfolio optimisation models based on two real data sets. An evaluation was conducted using Shannon's entropy and Gini-Simpson's entropy for portfolio selection. As Aksaraylı and Pala (2018) mention in their article, the proposed approach can be applied to portfolio models with high moments.

Fard et al. (2021) propose an effective method for estimating hedging error asymptotically using the maximum entropy estimator. It is found that the highest hedging error for options is based on a generalised jump-diffusion model with kernel bias. By maximising Shannon's entropy under moment constraints, they can calculate the value-at-risk of the hedging strategy and expect that there will be a shortfall in hedging error. However, it should be noted that the maximum entropy approach can be used despite the non-normality of the underlying return distribution to estimate the asymptotic distribution of the adjusted error.

A dynamic portfolio selection model based on entropy has been proposed by MacLean et al. (2022). There must be a wealth surplus that exceeds or equals the shortfall. There must be a probability that the shortfall will drop below a certain level. Therefore, the model must function. An empirical analysis of asset pricing is based on asset pricing tests. Under the observed Sharpe ratio and the return to entropy ratio, the results of their study show that the dynamic portfolio using the proposed strategy shows a significant improvement.

The MVSK model is extended by Gonçalves-Bradley et al. (2022) by analysing the skewness and kurtosis of the distribution. As part of their analysis, they also include an information entropy variable to measure asset information efficiency and diversity. Furthermore, they try to incorporate the high levels of uncertainty inherent in market returns into the models. An analysis examines the possibility of providing additional information to investors using a multi-objective portfolio model. Entropy was primarily used to rank assets because they developed a model that optimised information and transferred entropy. Their approach was to filter the holdings by evaluating the value, momentum, and amount of data following the Fama & French model. It was then decided which index components were most critical. The findings appear inconclusive, but it is possible to enhance model performance by incorporating a fuzzy framework.

Using a heterogeneous optimism and pessimism approach, Gong et al. (2022) study portfolio selection problems that include heterogeneous optimism, allowing uncertainty about future returns to be captured. When the market is in a turbulent phase, the entropy of equity market indexes decreases, so the indexes that earn will be more predictable and regular. In his study of elliptic and hyperbolic pseudo-entropies, Gupta (2022) investigates the concept of elliptic entropy. He uses adaptive indexes and oval and semi-entropic measures to capture investor attitudes. An optimisation problem involving coherent fuzzy numbers is used to use these risk measures. Taking the results of this study into account, he discusses how both linear and semi-entropy systems have advantages and disadvantages. It is argued that they are superior to other approaches in the literature by comparing them in different directions.

2.3 IET and risk measurement

Entropy is used in information theory to assess a message's degree of uncertainty and disorder (Shannon, 1948). Researchers such as Gulko (1999) and Dionisio et al. (2006) consider entropy a measure of information. Using this approach from statistical physics, Ausloos (1998) and Dacorogna (1999) quantify the disorder and uncertainty of dynamic systems.

Shannon's entropy (1948) measures how much information is present in a message instead of how much can be predicted. Statistical properties of a letter or word pair, triplets, or redundancy in language structure are examples of the latter. According to MacKay (2003), communication's "informational value" depends on its degree of surprise, defined in terms of entropy. An event that is highly likely to occur is communicated with basic information if it occurs. Alternatively, the message is much more informative if an unlikely event occurs. Knowing that a particular number will not win a lottery provides incomplete information since any chosen number is improbable. In contrast, knowing that a specific number will win a lottery has high informational value since it conveys an outcome of very low probability.

Several studies have found that stock prices are significantly modified by information about their fundamentals (Ross, 1989). As a result of the duality of their standard units, entropy is indexed with uncertainty and a lack of knowledge. A small quantity of data represents one possibility out of two, and a small amount of freedom represents one choice out of two. Shannon's memory is rehabilitated by entropy/information; independence/entropy considers fundamental non-determinism (Brissaud, 2005) on an entropy/information basis.

A method for measuring the knowledge base of an economy was proposed by Dolfsma and Leydesdorff (2008) using probabilistic entropy. Uncertainty reduction can be calculated as negative entropy with mutual information in three dimensions (or more). Several dimensions of a knowledge economy are crucial to its success, such as the size of firms, the location of companies, and the type of technology. A comprehensive dataset of all Dutch companies registered with the Chambers of Commerce can be used to refine well-known empirical findings for the geographical dimension.

The entropy model developed by Gibbs, Renyi, and Shannon has been used to study uncertainty in several ways. According to Dionisio et al. (2006), "entropy can be used to measure uncertainty in finance in several theoretical and empirical contexts". Entropy can be used both theoretically and empirically to measure uncertainty in finance. Physicists provide these operational tools because they provide diversity in uncertain situations. A variable is fully disordered when its entropy is maximised in a time series.

Reassessing the recent finding that no established portfolio outperforms a naively diversified portfolio, Behr et al. (2013) developed a constrained minimum-variance portfolio strategy. Minimal-variance portfolios have higher out-of-sample variance than diversified zero-variance portfolios. The portfolio strategy they use produces higher Sharpe ratios than 1/N. Sharpe ratios across our six empirical datasets increase by an average of 32.5%.

Yu et al. (2014) evaluate portfolio selections incorporating different entropy measures using multiple criteria methods. To enhance the feasibility of models, they show that models using Yager entropy outperform other models. When models include entropy, allocating assets without considering entropy is more feasible. Fund managers must handle Shannon or Yager, entropy-based diversification models.

Traditional risk and uncertainty measures are inadequate and ideal for identifying investment-related risks based on entropy. Mahmoud and Naoui (2017) stated that the power laws are advantageous in analysing uncertainty and asset value because they do not assume a normal distribution. The index reflects the firm's reality and needs to measure volatility, finance, and risk. Risk assessment and portfolio selection require a robust algorithm to apply entropy to risk assessment.

Carroll et al. (2017) evaluated minimum-variance allocation strategies for performance benefits based on time-varying asset correlations. The importance of mean-variance optimisation to portfolio weights has been well documented (Best & Grauer, 1992). Therefore, several papers use the global minimum variance portfolio to optimise portfolios without specifying assumptions about expected returns (Becker et al., 2015; Bodnar et al., 2017). Economic fundamentals are affected by stock market uncertainty, according to (Ahn et al., 2019). As a result of uncertainty shocks, industrial production declines for a short time, leading indicators to drop and rebound quickly, and systemic risk increases.

The time-frequency domain is a dynamic, bidirectional channel of causal information transmission (Dhifaoui et al., 2022). Ripple and Bitcoin transmit information bidirectionally; an investigation of information sharing between cryptocurrencies during the COVID-19 crisis was conducted by Assaf et al. (2022). According to Ünal (2022), COVID-19 causalities spread across 70 countries. Epidemiologists will benefit from his results as they portray COVID-19 spreading structure among countries. Transfer entropy (TE) outperforms traditional VAR methods based on the estimation of approximate entropy estimates during the COVID-19 era (Caferra, 2022).

3. Result and Discussion

This study critically examines several models developed after the Capital Asset Pricing Model (CAPM) in the 1970s, which are based on a relatively narrow hypothesis assuming financial markets operate in a Gaussian universe and are inherently efficient (Markowitz, 1952; Fama, 1970; Sharpe, 1964). However, empirical challenges to these models have surfaced over time, revealing their limitations in describing market behaviour due to their overly restrictive assumptions. As a result, alternative methods, such as power laws, have been considered to quantify better uncertainty surrounding future price movements.

According to Mandelbrot (1967, 1971) and Mandelbrot and Hudson (2004), these models cannot describe the reality of market behaviour since their hypotheses are too restrictive. Due to the lack of information captured by these models and the inability to quantify the uncertainty surrounding future price movements, it is necessary to refer to power laws. Thus, several explanations, including the market entropy approach, are interesting to consider, and it is worth considering several of them. As a result of these findings, research was directed towards reimagining explanations for market behaviour in conjunction with IET's research.

As a result, of a thorough search of the economic and financial literature, several works show entropy as a fundamental property of economics and finance. These works include those of Gulko (1999), who established an entropy theory. Researchers have turned to novel explanations of market behaviour. Financial-economic literature reviews reveal several works (Gulko, 1999; Mandelbrot, 1971). Although Fama (1970) argues that not all markets are efficient since not all markets enjoy pure and complete competition and frictionless transactions.

Financial instruments' prices fluctuate from one period to another in a manner known as a “random walk”. In so far as it implies that future price movements cannot be predicted, this idea is based on the work of Mandelbrot (1971), was highlighted by Samuelson (1973), and conditions the vision of efficiency testable degrees (strong, semi-strong, and weak).

Generally, entropy can be considered a measure of risk in portfolio returns as it is derived from the theory of information (Shannon, 1948). It has been shown that the amount of uncertainty/randomness in a probability distribution can be quantified by incorporating all higher-order moments (Cover & Thomas, 2006) and by considering the entire likelihood distribution when estimating the likelihood of an investment event.

Several studies support the claim that transfer entropy is more effective than other methods for evaluating causality. First, transfer entropy is a quantitative causality measure that detects nonlinear causal relationships between variables. Thirdly, (TE) may provide better results for identifying the relationship than traditional VAR methods (Caferra, 2022).

Several types of causality planes have been proposed in recent years that can be used to analyse one-time non-linear complex systems, such as the complexity-entropy causality plane, the Shannon-Fisher information plane, and the Renyi-Tsalli entropy plane. These planes are widely used in numerous fields. Despite this, applying these definitions to data sets containing two or more remarkable dimensions is not done daily. According to Wang and Shang (2021), dispersion entropy performs better when analysing complex systems. As part of the investor's request, the investor may be able to provide further information about the proposed framework using the Renyi effective transfer entropy approach, which is based on CEEMDAN.

A market price is considered efficient if it minimises investors' uncertainty about future price movement under the efficient market hypothesis. There is a strong case for the entropic market hypothesis that the entropy of consensus beliefs about future price changes is an adequate measure of collective uncertainty in the marketplace. As a result, entropy is maximised when the market becomes more efficient from an information perspective. Therefore, entropy provides a mechanism to interpret Gulko's (1999) work from an information theory perspective.

According to the entropy hypothesis of markets, a market with an informationally efficient price level should be characterised by market beliefs with maximum entropy. To identify Financial Market features due to non-linear dependencies, Barbi and Prataviera (2019) suggest using mutual information network analysis to determine the characteristics of the markets.

BPA uncertainty is measured by some scholars using entropy, a measure of randomness. Deng's (2016) entropy can effectively measure fundamental BPA uncertainty in several fields, like fuzzy multicriteria decision-making. It can be applied in many areas, such as BPA uncertainty assessment. A generalised Jenson-Shannon divergence approach based on belief functions was developed by Xiao (2020); its successful application to medical diagnostics has provided evidence of its validity and applicability.

Owing to the lack of consideration for the relationship between focal elements in Xiao (2020), divergence measurement for belief functions was proposed to measure BPA uncertainty. The work measured BPA uncertainty from the perspective of complex evidence distances.

Shannon entropy undoubtedly provides a novel perspective on uncertainty in probability theory. VarEvidence theory has various entropies, the primary uncertainty associated with basic probability assignment (BPA). Despite this, D spitfire is controversial regarding the measurement of uncertainty from a physics perspective and the basis of how these entropies are computed. As a result, the method for measuring BPA uncertainty is still open.

In probability theory, Shannon entropy provides a novel perspective for measuring uncertainty. In evidence theory, various entropies exist for measuring the primary tension of basic probability assignment (BPA). However, these entropies are controversial from the standpoint of uncertainty measurement and physics requirements. Therefore, the process for measuring BPA uncertainty currently remains an open issue in the literature.

It seems to us that the identification of entropy with a measure of knowledge (if we measure ignorance, then we measure ability) or of belief is based on a double title, formal between the mathematical formulas of Shannon's and Boltzmann's entropies and epistemic insofar as Shannon's insofar as the entropy of Shannon is comparable to information (in the technical sense that he gives it in his theory of communication). In contrast, as previously shown, Boltzmann's entropy can lead to an epistemic interpretation.

Our findings highlight the potential of Information Entropy Theory as a tool for measuring risk and valuing assets. With IET, we offer new opportunities to improve financial decision-making and understand market dynamics by addressing the limitations of traditional models and providing a more comprehensive perspective on market behaviour and uncertainty.

4. Conclusion and Remarks

This study examines empirical research on market efficiency, portfolio selection, and asset valuation to formulate a comprehensive theoretical framework of Information Entropy Theory (IET). Our goal is to determine how entropy can address the limitations of traditional models of financial asset valuation, such as the market efficiency hypothesis, the Capital Asset Pricing Model (CAPM), and the Black-Scholes option pricing model. We found several interesting properties of IET that facilitate portfolio selection and boost asset valuation. Specifically, this is the case when models are uncertain, loss functions are nonlinear, and the risk factors are not normally distributed.

We have conducted a literature survey that spans an extensive period, from 1948 to 2022, focusing on well-known asset pricing models and prominent IET research. Among portfolio managers, we find that they are deeply concerned about managing the valuations of their investments to maximise returns and minimise risk for their investors. Market information efficiency has been extensively studied to better understand financial markets' behaviour.

In conditions where return distributions deviate from Gaussian assumptions, the entropy-based portfolio selection model emerges as a preferred alternative to the standard model. It is important to note that this model provides richer information about the asset and its distribution probability. This method emphasises the diversification principle and relies more heavily on price information to make decisions. As a result, the entropic interpretation of the no-arbitrage principle, which allows for extreme variations, makes it more appropriate for valuing derivative instruments than the Gaussian distribution because it accounts for extreme variations.

Through its successful use, the practical reliability and efficiency of IET become evident. It is more appropriate to use entropy instead of variance when dealing with non-Gaussian return distributions, and power laws are more suitable for analysing uncertainty and asset values because they do not rely on standard distribution assumptions. Considering Markowitz's work in light of a better understanding of uncertainty, this revaluation of Markowitz's work gives rise to a mean-entropy approach as an alternative perspective to the traditional mean-variance approach to asset selection.

Future research could build on existing literature by looking at the relationship between IET and the scores related to the environment, social issues, and governance (ESG). Furthermore, by revisiting classical valuation models based on entropy theory, we could gain valuable insight into the inefficiencies of financial markets when it comes to providing information to investors. From a perspective of entropy, this is the result of an analysis. It should be noted that the entropy approach emphasises decomposing total risk into systematic and unsystematic risks through statistical methods when measuring the risk associated with financial assets.

As a result, the findings of this study illustrate the importance of Information Entropy Theory as a tool for overcoming the limitations of traditional asset valuation models. Portfolio managers and investors can benefit from incorporating entropy-based perspectives when managing uncertain environments, as investors can make more informed decisions in pursuit of optimal returns while mitigating risk by incorporating entropy-based perspectives. Research in this area is expected to enrich our understanding of financial markets in the future and refine the tools used to measure and value assets and risk in the future.

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